Journal of Civil Engineering and Vocational Education

EISSN: 2622-6774
Vol 10 No.3 September 2023
http://cived.ppj.unp.ac.id/index.php/CIVED

Design and Analysis of Beam with Opening using Strut and Tie Model

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Received 9th August 2023; Revision 25th August 2023; Accepted 17th September 2023

ABSTRACT

Concrete beams on structures sometimes have to be covered for utility lines. For the case of concrete beams with openings, special calculations must be made because the lengths of the dimensions no longer have the same dimensions. One of the approaches is with the strut and tie model. Currently, there are no standard provisions in the manufacture of the strut and tie model, as it is an approach to calculating in areas where Bernoulli's law does not apply. In order to facilitate the creation of a strut and tie model, the study used a simple software tool called BESO2D. The number of samples of the test model is 4 variations with different position and size of the hole. The dimensions of the beam used are 150x300 mm with a length of 2000 mm. Each of the beams will be given a reinforced based on the working load and from the strut and tie model frame. The behavior of each beam will be analyzed using the software of the finite element method to specific concrete named ATENA.

Keywords: Beam With Opening; Strut and Tie Model; Finite Element Method; BESO2D.

INTRODUCTION

Parts of structural elements such as beams in a reinforced concrete building sometimes have to be perforated for utility lines. The holes in the beams are in fact often not taken into account in the planning design, because initially the beams are designed in full beam without holes. However, due to the need for utility lines during the execution work, and for the reason that they should not be exposed due to aesthetic factors, the beams are usually perforated without any calculation analysis and special handling in the execution. A small hole in a beam may not have a major effect on the strength of the concrete structure depending on the percentage of the size and number of holes to the beam dimensions, but if the hole is large enough for the purpose of a utility pipe line as shown in Figure 1, it is likely to cause an imbalance in the flow of forces and stress concentrations due to changes in the geometry of the dimensions of the beam structure.

In the case of hollow beams, a strut and tie model is recommended. The strut and tie model is based on the flow of forces that occur in the structure, which are passed on to the support. In this case, of course, imagining the flow of forces that occur will be difficult and may not be able to be imagined, especially if you have to make a strut and tie truss model. So the trial and error strut and tie truss model is done several times to get the optimal model shape based on the flow of forces that occur due to the working load.

CIVED
Journal of Civil Engineering and Vocational Education

EISSN: 2622-6774
Vol 10 No.3 September 2023
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Figure 1: Example of a reinforced concrete beam perforated by a utility pipeline

MATERIALS AND METHODS

Basis for Calculation of Nominal Flexural Strength of Beams

Reinforced concrete is a combination of two elements, namely concrete elements (strong compressive resistance weak to tensile) and reinforcing steel (strong resistance to tensile and compressive). So that the behavior of these two elements when working together shows *nonlinear* behavior. However, to simplify the calculation, the nominal flexural strength of the cross section is assumed to be reached when the strain at the time of the outermost compressive fiber reaches the limit strain of the concrete, which is 0.003 as shown in Figure 2 as follows:

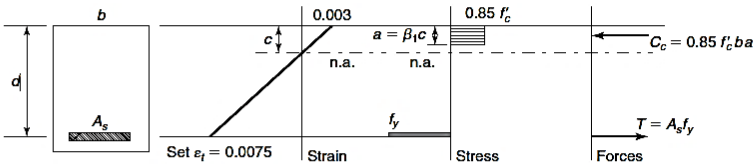

Figure 0: Assumptions for single reinforcement design of concrete beams

The basic assumptions in the theory of bending in square concrete beams are in accordance with the concrete SNI:

- A cross-section perpendicular to the bending axis that was flat before bending shall remain flat after bending (article 10.2.2).
- No slip between concrete and steel reinforcement (at the same level, the strain in concrete is the same as the strain in steel) (article 10.2.2).
- The stresses in concrete and reinforcement can be calculated from the strains using the stress-strain relationship of concrete and steel (article 10.2.4).
- For the calculation of the flexural strength of cross-sections, the tensile strength of concrete is neglected (article 10.2.5).
- Concrete is assumed to collapse when its compressive strain reaches the limit strainε cu =0.003
- The stress-strain relationship of concrete can be assumed to be square, trapezoidal or parabolic (clause 10.2.6).

SNI Beton article 10.2.7 allows the use of an equivalent square compressive stress distribution for the ultimate calculation of cross-sections. The square stress block is defined as follows (Imran, 2014):

- The uniform compressive stress ofα 1fc' (whereα 1=0.85) is assumed to act along the equivalent compressive zone that is a=β 1c away from the outermost (extreme) compressive fiber.
- The distance c is determined from the position of the outermost compressive fiber to the

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EISSN: 2622-6774
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neutral axis measured perpendicular to that neutral axis.
- The value ofβ 1 is taken as follows:
For fc' < 30 MPa → β 1 = 0.85
For fc' < fc' < 55 MPa → β 1 = 0.85-0.008 (fc'-30)
For fc' > 55 MPa → β 1 = 0.65
In this case, only two parameters, d andβ , need to be used for reinforcement design calculation to describe the equivalent square compressive stress block. Based on the stress distribution, the flexural strength can be calculated with equations (1), (2), (3) and (4) as follows:

$$C = 0.85\ fc'\ ab \qquad (1)$$

$$T = As\ fy \qquad (2)$$

Assuming the reinforcement has yielded before the concrete reaches its compressive limit strain.
Equilibrium conditions→ C = T then,

$$a = \frac{As\ fy}{0.85\ fc'b} \qquad (3)$$

So,

$$Mn = As\ fy\left(d\frac{a}{2}\right) = Asfy\left(d - 0.59\frac{Asfy}{fc'b}\right) \qquad (4)$$

Equations (1), (2), (3) and (4) apply only to ordinary reinforced concrete beams without any holes and are also not a category of high beams.

Basic Concept of Strut and Tie Model

Research on reinforced concrete continues to grow today. Reinforcement calculation approach methods such as the *Strut & Tie Model* are not a new science. The method evolved from the "*Truss Analogy Model*" first introduced by Ritter (1899) and Mörsh (1902) which was obtained based on observations of crack patterns that occurred in experimental testing of reinforced concrete beams as shown in Figure 3. From the observed crack pattern, it is imagined to be a form of truss structure pattern shown in Figure 4.

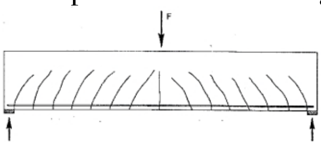

Figure 3: Crack pattern in a beam due to F load (moment and latitudinal force)

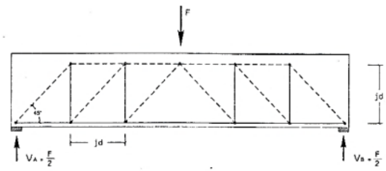

Figure 4: Truss analogy for reinforced concrete beams according to Mörsch

The truss model proposed by Mörsh is a typical truss consisting of tensile and compressive

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bars. Diagonal compressive bars are used at an angle of 45° where the compressive force on these bars is borne by the concrete itself, while vertical tensile bars are shear reinforcement installed to resist latitudinal forces. As with the reinforcement of ordinary beams, the horizontal compressive bars and parallel tensile bars are the forces required to bear the bending moment. The design of the "Truss Analogy Model" has not been able to cover all structural elements that have complicated geometries, or beam structures that have openings or holes, high beams, so the "*Truss Analogy Model*" was further researched by experts, namely Rüsh (1964), Kupfer (1964), Leonhardt (1965), Elfgren (1972), Müeller (1978), Collins and Mitchell (1980), Thürlimann et. al.al (1983), Hsu (1982), Marti (1985) and MacGregor (1986) and other researchers. Then the Truss Analogy Model was developed by Schlaich, Schäfer and Jennewein (1982-1993) into a more general and consistent form which became known as the "*Strut and Tie Model*" or the support and *tie* model by dividing the structure into two regions, namely the D and B regions. The use of the "*Strut and Tie Model*" can be applied to various geometrically discontinuous structures, beams or walls with openings, and other complex structures.

Region D and Region B

The design of concrete structures as in general consists of two regions, namely region D and region B. (Schlaich et al., 1988) has built a basic design philosophy that is consistent in structures consisting of regions D and B (see Figure 5), namely design with *strut and tie models*. However, the *strut and tie model is* mostly applied to the D region, while the B region is more devoted to the design of shear and torsion effects. The application of the strut *and tie model* in the design of concrete structures begins with the determination of regions D and B.

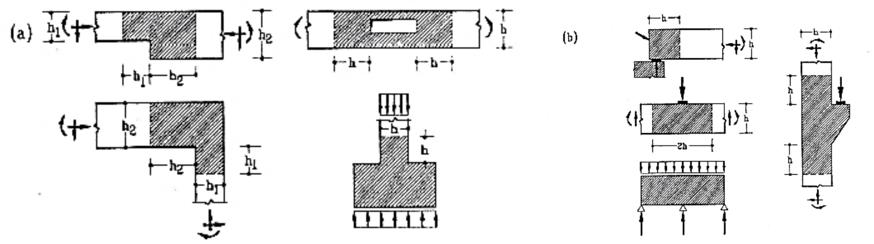

Figure 5: Region D (shaded area) with non-linear stress distribution due to (a) geometric change of the structure (b) loading effect of the structure (Schlaich et., 1987).

The two areas describe the *load path* as a transfer of force that occurs in concrete structures in a cracked state due to loads that occur from the source of force to the support (Hardjasaputra and Tumilar, 2002).

Strut and Tie Planning (SNI 2847-2013)

The strut and tie design and nodal zone are calculated based on equation (5):

$$\emptyset F_n \geq F_u \quad (5)$$

Where:

Fu is the factored force acting on the *strut*, in the fastener, or on one face of the *nodal zone*,
Fn is the nominal strength of the *strut*, *tie*, or *nodal zone.*
Ø is the strength reduction factor

Strut Compressive Strength

The compressive strength of the strut without longitudinal reinforcement shall be taken as less than the following equation (6):

Fns = fce Acs (6)

Where:
Acs is the cross-sectional area at one end of the strut
fce is the effective compressive strength of concrete in the strut should be taken as equation (7):

fce = 0.85 βs fc′ (7)

For the compressive region, it is shown in equation (8):

Acs = bw Ws (8)

For the value of βs taken according to SNI 2847-2013 appendix A2.2.1-3 is as follows:
βs = 1 for struts with a prism shape.
βs = 0.75 for struts that can very likely be idealized as bottle shapes, such as diagonal struts in high beams or consoles
βs = 0.4 for struts that have diagonal cracks in the span, which can significantly weaken the strut.

Tie Strength

In concrete structures the tensile bars may be one or a collection of ordinary reinforcing steel or may be one or a collection of properly anchored prestressed tendons. The strut and tie strengths are applied to reinforced concrete structures in the limit state, so in the service condition the crack width of the tie pull rod needs to be checked by crack width limitation. The nominal tie strength Fnt should be taken as equation (9):

Fnt = Ats fy + (fse + Δ fp) (9)

Where:
Fnt = nominal strength of the fastener (kN)
Ats = reinforcement area (mm2)
Atp = prestressed tendon steel area (mm2)
Fy = steel grade (MPa)
Fse = effective stress loss in prestressed tendon steel (MPa)
Δfp = addition of prestress force in addition to level load (fse +Δ fp)

Nodal Zone Strength

Nodal is the meeting point of three or more bar forces from strut, tie, centered load, pedestal reaction with various variations (Hardjasaputra, 2016). In general, it can be divided into four types of meeting joints, namely:
a. CCC-nodal means that there are three compressive forces meeting at the nodal.
b. CCT-nodal means that two compressive and one tensile forces meet at the nodal.
c. CTT-nodal means that one compressive and two tensile forces meet at the nodal.
d. TTT-nodal means that there are three tensile forces that meet at the nodal

Collins and Mittchell (in Harjdasaputra, 2016) state that strut width is influenced by the stiffness of the reinforcing tensile bars anchored by stirrups. When the dimensions and geometry of the nodal have been determined, the nominal compressive strength of the Fnn connection point region (nodal) is described in equation (10) as follows:

$$Fnn = fce\ Anz \qquad (10)$$

Where:
fce is the effective compressive strength of concrete at the nodal region.
Anz is the smaller of:
- Area in front of the nodal region where Fu acts, taken perpendicular to the line of action of Fu
- The cross-sectional area passing through the nodal region, taken perpendicular to the line of action of the resultant force on the cross-section.

When restraint reinforcement is provided in the nodal region and its effect is supported by tests and analysis, the calculated effective compressive stress, fce at the face of the connection point region due to compressive and tensile forces shall not exceed the value given by equation (11):

$$fce = 0.85\ fc'\ \beta n \qquad (11)$$

The value of βn is determined by the type of meeting point based on SNI 2847-2013 appendix A.5.2.1-3, namely:
For CCC nodal zone βn = 1
For CCT nodal zone βn = 0.8
For CTT nodal zone βn =0.6
The reinforcement installation width can be determined according to ACI1318-2011 Appendix RA 4.2, equation (16):

$$\mathbf{W_t} = \frac{\mathbf{F_{nt}}}{(\mathbf{f_{ce}} \times \mathbf{b_s})} \qquad (11)$$

***Strut and Tie Model* Evolution Using BESO2D (*Bi-Directional Evolutionary Structural Optimization*)**

BESO2D is a software to make it easier to get a Strut and Tie frame shape model on structural elements based on the method of eliminating inefficient structural material criteria little by little until a truss structure is formed as shown in Figure 6.
The process of this optimal evolution of the STM structure in the BESO2D program can be explained as follows:
- Eliminate materials that are inefficient in carrying loads
- Elements that are removed are only those that have a low degree of sensitivity to deformation
- In the topology optimization stage of STM, material removal can be adjusted and can be repeated until it is in its most optimal form.
- The accuracy of the solution based on the Element Removal Ratio (ERR) is small and an ERR percentage of 1% or 2% is recommended.

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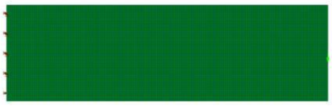


(a) (b)

(C)

Figure 6: BESO2D Strut and Tie Model Optimization Process. (a) *Meshing* of cantilever beam structural elements with supports and loads (b) Process during STM optimization with 80% volume fraction (c) Final STM optimization process with 35% volume fraction.

Test Model Specifications

The quality of concrete used is 30 MPa, the quality of reinforcing steel is 400 MPa, the load acting in the same position on each beam is two point loads at mid-span and is assumed to be 50 kN to get the reinforcement configuration of the *strut* and *tie* model.

The hollow reinforced concrete beam test model has a cross-sectional dimension of 150x300 mm with a span length from support to support of 2000 mm. The test model is a simple beam with a roller-joint support. The test model variations are shown in Figures 7, 8, 9, 10, and 11 as follows:

1.)BTL beams are concrete beams without holes with dimensions 150x300 mm with a beam span of 2000 mm.

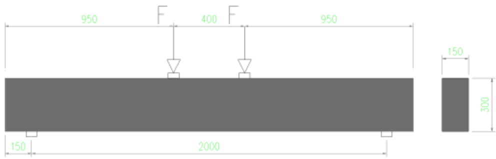

Figure 7: BTL beam

2.)LTB1 beams are beams with a hole position in the center of the span (critical bending region) with a size of 100x100 mm and only one hole.

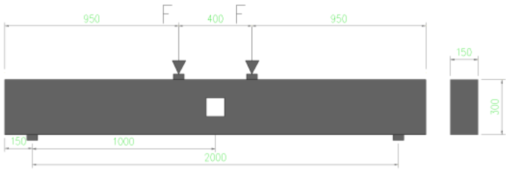

Figure 8: LTB1 beam

3.)LTB2 beams are beams with a hole position in the middle of the span (critical bending area) with a hole size of 200x200 mm.

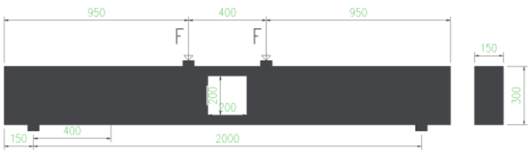

Figure 9: LTB2 beam

3.)LPB1 beam is a beam with a hole position between the support (critical shear region) and the load position with 1 hole size 100x100 mm each on the left and right side of the beam.

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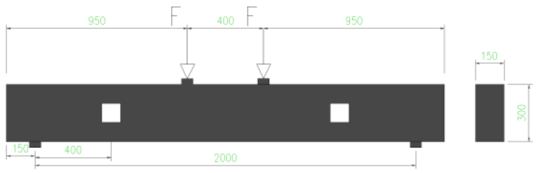

Figure 10: LPB1 beam

4.)LPB2 beams are beams with holes positioned between supports (critical shear region) with a hole size of 200x200 mm.

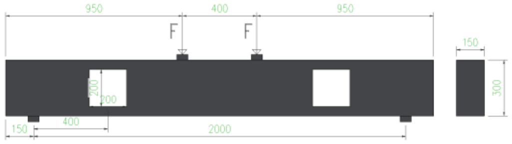

Figure 11: LPB2 beam

Table 1: Test model variations

No.	**Test Model**	**Cross-sectional Dimension (mm)**	**Beam Length (mm)**	**Hole Size (mm)**
1	BTL	150x300	2000	No holes
2	LTB1			1@100x100
3	LTB2			1@200x200
4	LPB1			1@100x100
5	LPB2			1@200x200

RESULTS AND DISCUSSION

Determination of Optimal Topology of Strut and Tie Truss with BESO2D *Software*

The method in this *software is* developed based on the criterion of removing material components from the structure so as to form a truss model based on the load and support position. The parameter items in this software can be preset and can be used to monitor the topology optimization process of the strut and tie frame. The following figures 12, 13, 14, 15, 16 are the topology of the strut and tie model resulting from BESO2D in the variation of each hollow beam.

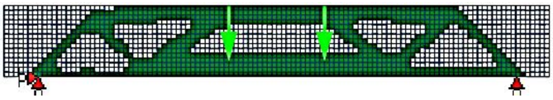

Figure 12: BTL Strut & Tie Beam Model

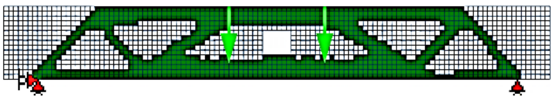

Figure 13: Strut & Tie model of LTB1 beam

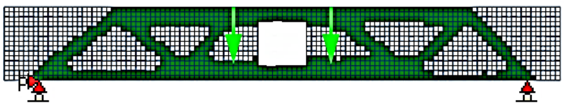

Figure 14: Strut & Tie model of LTB2 beam

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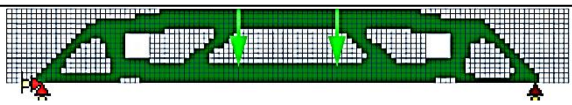

Figure 15: Strut & Tie model of LPB1 beam

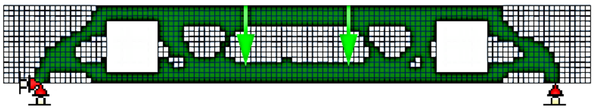

Figure 16: Strut & Tie model of LPB2 beam

From the results of the truss model produced on concrete beams with variations in hole position and size, it can be seen that visually the truss configuration is not too much different. The truss model does not fulfill the concept of a stable truss in engineering mechanics principles, but in the Strut & Tie Model, the truss shape does not have to be stable in mechanics principles as the truss made by Maxwell & Breen (2000) who applied the strut and tie model to experimental tests of hollow high beams (see Figure 4.6).

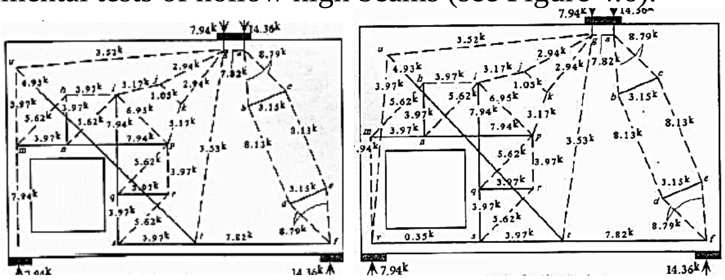

Figure 17: Strut and Tie Frame Model by Maxwell & Breen

Although mechanically unstable, the frame model must still fulfill the equilibrium of the force flow point from the source of force to the support, so in this case it is necessary to adjust the shape of the frame.

Calculation of STM Rod Forces with SAP 2000

The STM truss model generated from BESO2D is simplified and modified in such a way as to achieve a truss shape that has a balance of force flow points from the source of force to the support so that it can be analyzed in the calculation of tensile and compressive forces with structural analysis software, SAP 2000. This program can analyze unstable frames, provided that the equilibrium of the force flow point from the force source to the support is met.

Table 2: Resume of trunk forces of the test model

No	Nama Model uji	Gaya tarik maksimum(kN)	Gaya tekan maksimum(kN)	Gaya Tarik diagonal(kN)
1	BTL	160	160	64.03
2	LTB1	160	160	64.03
3	LTB2	160	160	64.03
4	LPB1	160	160	53.9
5	LPB2	133.33	250	206

Single Reinforcement Design

Reinforcement is calculated based on equation (5) tie design requirementsϕ Fnt $\geq$ Fut, the nominal tensile force of the tie is Fnt = Ats fy reduction factorϕ =0.75 and fy=400 MPa.
The reinforcement design resume based on the maximum tensile bar force on each test beam

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model is as follows:

Table 3: Resume of tensile reinforcement amount of the test model

No	Nama Model uji	Gaya tarik maksimum(kN)	Luas Tulangan(mm2	Tulangan Yang Digunakan
1	BTL	160	533	4D13
2	LTB1	160	533	4D13
3	LTB2	160	533	4D13
4	LPB1	160	533	4D13
5	LPB2	133.33	444	4D13

From table 3 above, it can be seen that the area of flexural reinforcement required based on STM is relatively the same, so the single reinforcement used for all beams in the test model is also the same, namely 4D13 as shown in figure 4.12 below:

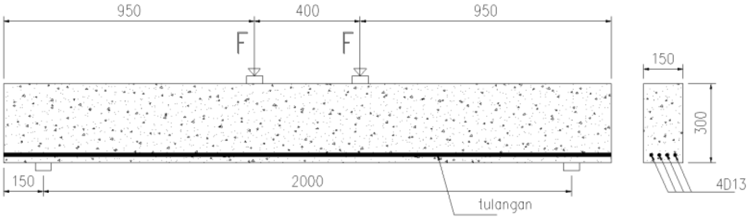

Figure 18: Single Reinforcement Design of Concrete Beam

Finite Element Modeling of Test Model with ATENA Software

The concrete used in this analysis is with a quality of 30 MPa, then a *two point* load simulation is carried out using ATENA *software* with the addition of each step is $5x10^{-5}$ mm *deformation* (*prescribed deformation*) and carried out 100 times the deformation step. The results can be shown as in Figure 19, up to Figure 30

Crack Pattern of Concrete Beam with Single Reinforcement

1.)BTL beams

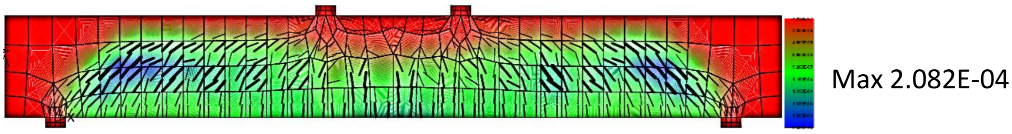

Figure 1**:** Crack pattern of BTL beam

2.)LTB1 beam

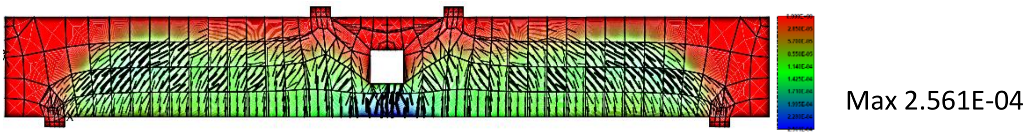

Figure 2: Crack pattern of LTB1 beam

3.)LTB2 beam

Image **3**: Crack pattern of LTB2 beam

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4.)LPB1 beam

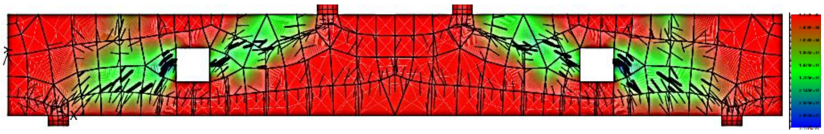

Figure 4: Crack pattern of LPB1 beam

5.)LPB2 beam

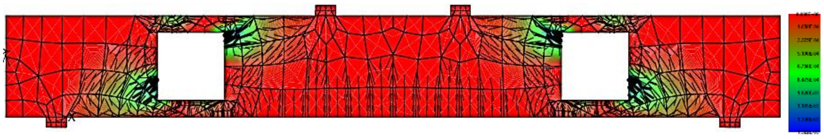

Figure 5: Crack pattern of LPB2 beam

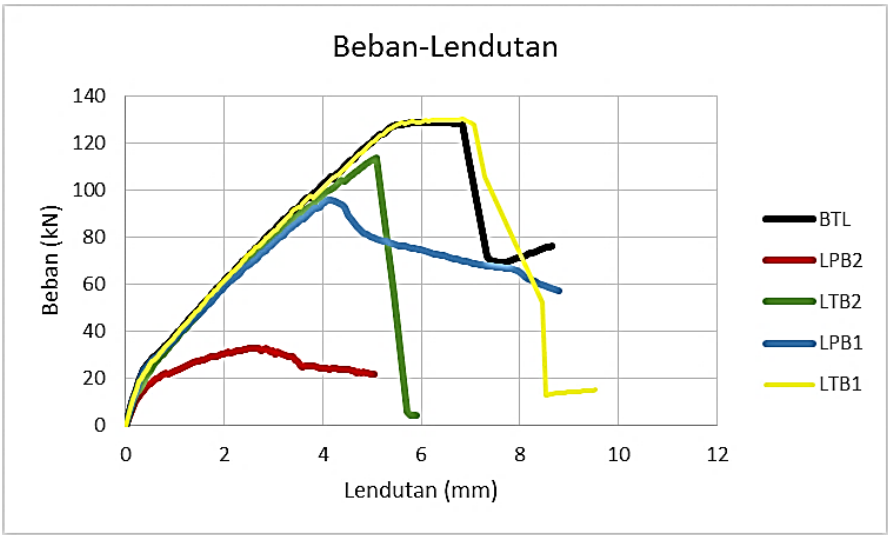

Figure 6: Load-deflection comparison graph of beams with single reinforcement

Based on Figure 24 load-deflection curves above, it can be concluded that the behavior of reinforced concrete beams (single reinforcement) that are given openings in the critical moment region with the percentage of holes to beam height < 33% shows almost the same behavior as intact beams. However, if openings with the same percentage are placed in the critical shear region, the maximum capacity of reinforced concrete beams (single reinforcement) is reduced quite drastically.

Reinforcement Design for Hole Opening Region

1.)BTL beams

This beam test model serves as a reference in the design of extra reinforcement in beams with openings.

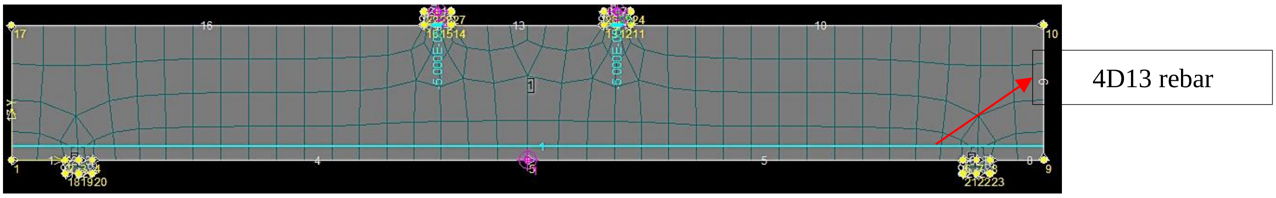

Figure 1 BTL reinforcement design

2.)LTB1 beam

Since the deflection load graph for beams with a 100x100 mm hole in the middle, there is a

CIVED Journal of Civil Engineering and Vocational Education
EISSN: 2622-6774
Vol 10 No.3 September 2023
http://cived.ppj.unp.ac.id/index.php/CIVED

behavioral equation, there is no need to add extra reinforcement to the area with the opening, so a single reinforcement as in BTL beams is sufficient.

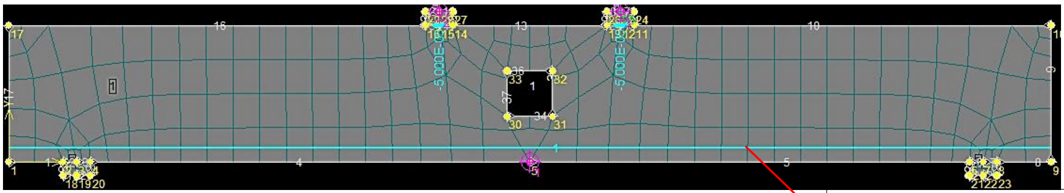

Figure **2**: LTB1 reinforcement design

3.)LTB2 beam

Beams with a 200x200 mm opening in the middle of this span need to be given additional reinforcement. From the results of the tensile force on the diagonal bars, the STM formula obtained the required reinforcement area is 213.43 mm2, so 2D13 is used.

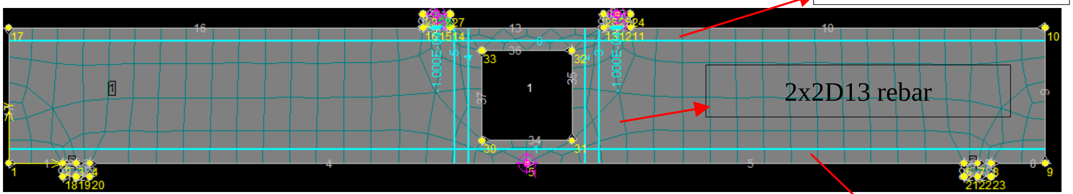

Figure 3: LTB2 reinforcement design

4.)LPB1 beam

Beams with 100x100 mm openings at the edge of this span need to be given additional reinforcement. From the results of the tensile force on the diagonal bars, the STM formula obtained the required reinforcement area is 179.67 mm2, so 2D13 is used.

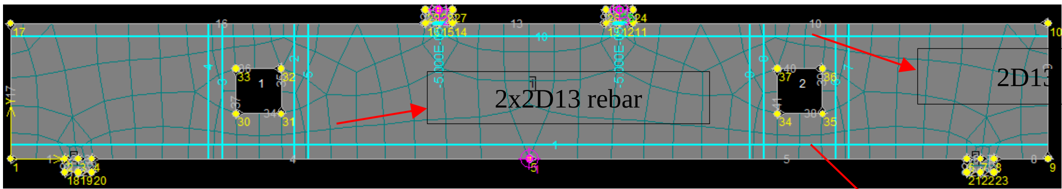

Figure 4: LPB1 reinforcement design

5.)LPB2 beam

Beams with 200x200 mm openings at the edge of this span need to be given additional reinforcement. The STM frame pattern on this beam is quite complicated and produces a large enough tensile rod force in the area close to the opening of 206 kN, so from the results of the tensile force on the diagonal rod, the STM formula obtained the required reinforcement area is 686 mm2, so 6D13 is used.

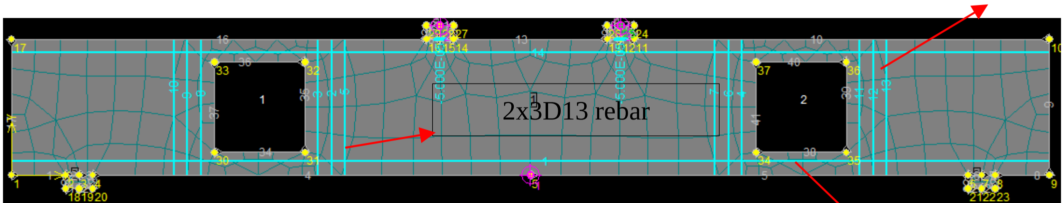

Figure 5: LPB2 reinforcement design

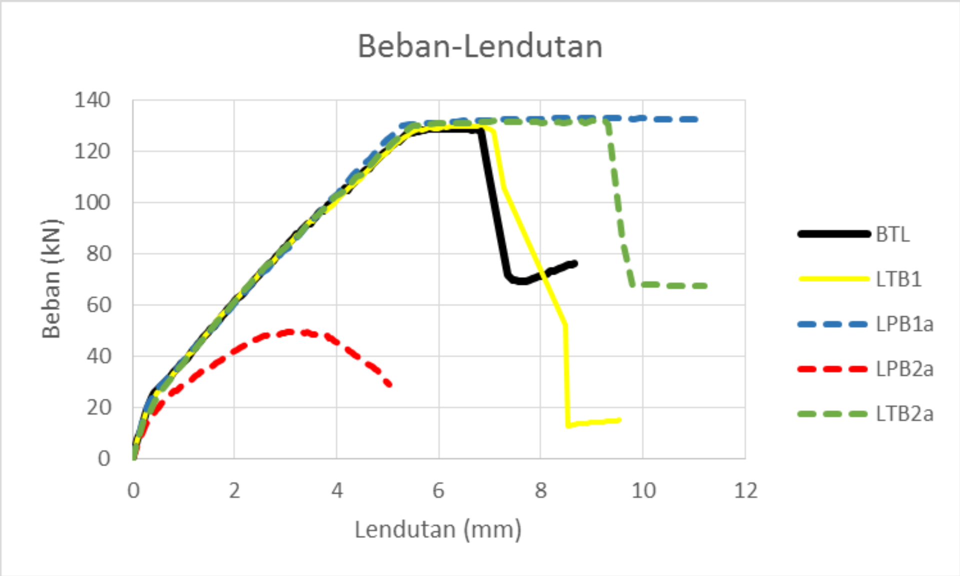

Figure 6: Load-deflection comparison graph of beams with extra reinforcement

Based on Figure 30 load-deflection curves above, it can be concluded that the behavior of reinforced concrete beams with openings up to 200x200 mm in the mid-span area (critical moment), can be accommodated by the addition of extra reinforcement based on the tensile force of the STM frame results, so that they have almost the same behavior as beams without holes. While at the support area, the addition of extra reinforcement can only be accommodated at a hole size of 100x100 mm. The hole size of 200x200 mm at the pedestal area even though extra reinforcement has been added in the opening area, it is still not able to approach the behavior of concrete beams without holes so that punching holes in the pedestal area with a relatively large size is not recommended.

CONCLUSION

With variations in the size of the holes in the concrete beam, as well as the position of the holes, the single reinforcement design of the strut and tie model method results in relatively the same reinforcement area, so for the flexural region using 4D13.

Behavior of unreinforced concrete beams:
1. Concrete with 100x100 mm openings of 300 mm beam height relatively does not affect the crack location.
2. For relatively small holes (100 mm of the 300 mm beam height), concrete beams without reinforcement behave almost the same as intact beams without holes.
3. The crack position has a major effect on unreinforced perforated concrete beams with 200 mm holes of 300 mm beam height.

Behavior of concrete beams with single reinforcement (strut and tie model results):
1. The location of cracks in perforated single reinforced concrete beams occurs at positions based on the holes where there is structural weakening. However, for beams without holes (with a single reinforcement) cracking occurs predominantly in the critical shear region.
2. At the ultimate condition, with the same hole size, the crack width is larger for holes in the critical shear region than for holes in the critical moment region.

3. Structural weakening by perforating the critical shear region is more dominant in reducing the ultimate capacity of single reinforced concrete beams than perforating concrete beams in the critical moment region.

Behavior of concrete beams with extra reinforcement (strut and tie model results)
1. For beams with hole conditions in the critical moment region, the addition of extra reinforcement in the area near the hole which is the result of the STM bar force is able to accommodate the weakening of the concrete structure due to hole openings up to 200x200mm.
2. For beams with hole openings in the critical shear region, the addition of extra reinforcement from the STM bar force results can only accommodate hole openings of 100x100mm.
3. Making relatively large hole openings in beams is not recommended in critical shear areas, even with additional reinforcement.

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